# Planning of heavy-duty vehicle platoon formulation: basic scheduling problem considering travel time variance

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## Abstract

Platooning of heavy-duty vehicles (HDVs) on highway has attracted extensive research attention due to its potential benefits on reduced fuel consumption, increased road capacity, enhanced traffic safety and so on. In reality, the departure time of HDV should be planned in advance concerning transport economic efficiency. The introduction of platooning opportunity requires coordination of the time tables or schedules of different HDVs and hence has direct impacts on the final planning decision. However, this problem has been barely explored. This study formulates the basic scheduling problem with platooning as one of the decisional variables, and analyzes optimal decisions for the simple network topologies. Numerical examples are given to illustrate the impacts of relevant parameters on the platooning opportunity such as the travel time variance and preferred arrival time difference. The results give useful insights about the platooning decision.

*Keywords*: Heavy-duty vehicle platoon; scheduling; travel-time uncertainty.

## 1 Introduction

In the last decades, many studies have reported the potential benefits of HDV platooning for transport system. It may reduce the air drag resistance and provide economic and ecological benefits including fuel saving, more efficient utilization of the road infrastructure and better traffic safety. The experiment conducted by [2] shows that the following truck driving at a distance of 10m could, compared to isolated driving, save fuel up to 20%. In the mean time, the leading truck also experienced reduction on fuel consumption. It is known that vehicle $CO_2$ emission is directly proportional to the fuel consumption. [13] report a 2.1% $CO_2$ reduction on expressway when the gap is 10m with a 40% penetration of platooning of heavy trucks by simulation. The result is even higher with shorter inter-vehicle distance. The implementation of platoon, especially on a large scale, emerges from the background of intelligent transportation systems. Since the concept of platooning originally proposed in [15] by California Partners for Advanced Transportation TecHnology (PATH), researchers and manufactures take advantage of the innovation and technological development in wireless communication and automatic control to analyze the technical feasibility of platooning regarding various focuses. Remarkably, Adaptive Cruise Control (ACC) and Cooperative Adaptive Cruise Control (CACC) enable more accurate maneuverability during platooning, for instance, merging, splitting and lane changing.

Previous research on platoon formation and coordination can be classified into two categories by the scope of the problem. One is vehicle sorting, merging and splitting in a node resolution or at highway on-ramps and off-ramps. Hall and Chin [7] concentrate on the formation and characteristics of platoon on ramps, define a set of platoon formation strategies and develop analytical models for performance measures. In the study, vehicles are assumed to be grouped by lanes and will

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remain isolated driving state once it splits from its platoon. Since the vehicles are not permitted to sort on highways, the average waiting time at entrance is considerable. [4] present an approach for platoon decentralized control. The introduction of a multi-agent system provides both longitudinal and lateral control and merge and split capabilities. By simulation with the MaDKit (the Multi-agent Development Kit) platform, the control strategy shows flexibility with respect to obstacle avoidance and reliability. Similarly, [17] also propose a higher-level longitudinal decentralized control algorithm targeting at safety, highway capacity and energy efficiency improvement. The other one is platoon routing and coordination on large-scale network. From the perspective of minimizing fuel consumption in platoon coordination, not much has been done. In general, there are two types of platooning modes based on the numbers of vehicles participate in the system, namely planned mode and temporary mode ([16]). Planned mode refers to vehicles with dedicated transportation assignments find their shared paths with other vehicles in advance and are supposed to get to the merging point (the start point of shared path) in time and split at the splitting point. While in temporary mode, vehicles search for their candidates for platoon in real time by means of vehicle-to-vehicle communication especially when approaching an intersection. This strategy is extremely suitable for large-number situation. [8] develop a "Local Route and Platooning Coordinator" (LRPC) for heavy-duty vehicles approaching the same node. The controller compares the fuel consumption following shortest path and local control strategy, if the additional fuel required to form the platoon can be compensated by the savings the vehicles will incur in the platoon, then the controller will implement platoon formation. Simulation on the simplified German autobahn road network, composed of 647 nodes, 695 edges and 12 destinations, shows the total fuel consumption has been decreased by 2-2.5% for 300 trucks. Furthermore, savings of over 6% can be observed for 2000 trucks. If the drivers are willing to accept slightly longer travel time, even more savings can be achieved. [11] assume if several vehicles enter the same link within a time interval, then those vehicles platoon during the whole link. By taking different values of time interval, the fuel savings fluctuate dramatically. This coordination is referred to as transport coordination, which gives a different perspective although it is inconsistent with practice. Global controller trying to coordinate every vehicle in a large-scale network is computationally intractable. [9] present integer linear programming formulations for large-scale, real-world platooning problem without considering deadlines. Besides, the performances of several heuristics are compared. The work reveals that savings of 9% are obtained for 200 trucks if all trucks share the same origin. Our work resides in the second category, i.e., platooning coordination in the traffic network and we try to make it clear for decision makers to consider the factors which will impact platooning benefits.

While the current HDV platooning studies are mainly driven by the technical requirement on autonomous driving, there is little research effort on freight transport planning when platoon becomes a decisional variable. Indeed, the introduction of HDV platoon increases the complexity of the transport planning problem such as fleet scheduling. So the main objective of this study is to figure out how and when it is beneficial to coordinate heavy-duty vehicles to form platoons, considering the travel time variance, mainly based on vehicle speed and preferred arrival time. In the context of transport economics, various aspects should be considered to measure the total cost, including customer service level, waiting time, total travel time, fuel cost and environmental cost, among which travel time and fuel cost are interrelated. As is known to all, travel time variance is related to congestion, which will cause additional fuel consumption and emission. Moreover, longer travel time is undesirable for drivers and a truck will not undertake other tasks until it finishes the ongoing journey. The value of travel time variance (or reliability) has attracted great attention in recent years. For example, [14] explored when travel time should be considered unreliable and how the (un)reliability can be measured and monitored in a traffic network. [5] formulated a scheduling problem and studied the impact of mean and standard deviation of trip durations on the utility, especially when the mean and standard deviation are linear functions of departure time. The reader may refer to [3] for a systematic review of the current state of research in travel time reliability. As mentioned above, by driving closely to each other, the trucks in the platoon will save fuel and thus reduce the fuel cost, i.e., the trucks should either depart from the origin at the same time or meet at the merging point and drive together on the shared paths. Aware of this, the platoon coordination problem becomes a scheduling problem in essence.

The rest of the paper is organized as follows. The next section presents a general description of the platoon scheduling problem and the basic approach of our model framework as well as the estimation method of different costs. In section 3, the common route problem is formulated and both analytical solutions and numerical examples are given to illustrate the impacts of different parameters. In section 4, the problem is extended to a simple network and more parameters are introduced and analyzed. Conclusions follow in section 5.

# 2    Problem formulation

## 2.1    Basic approach

In freight transport planning, HDVs are often assigned to travel from their origins, e.g. freight terminals, to destinations, e.g. city logistic centers or customers, with predefined time table. Given the origin, destination and preferred arrival time of each HDV, the fleet management system often needs to decide the departure time and even route choice for each vehicle so that

the transport cost can be minimized. The transport cost may contains several aspects, including fuel cost, driving cost and penalty on schedule miss. Depending on the type of goods that the vehicle carries as well as the customer's requirement, a schedule miss may lead to penalty if vehicle arrives early or late. Analytically, the total cost can be represented by

$$\mathbf{C}_T(\theta) = \mathbf{C}_F(\theta) + \mathbf{C}_V(\theta) + \mathbf{C}_D(\theta) \tag{1}$$

where $\mathbf{C}_F$ is the fuel cost; $\mathbf{C}_V$ is the driving cost; $\mathbf{C}_D$ is the penalty on schedule miss; $\theta$ represents a set of decisional variables for optimal transport cost such as departure time and route choice. While formulation of HDV platoon could reduce fuel consumption and transport cost, it becomes an additional factor that should be considered in transport planning.

Road traffic condition will obviously affect the freight transport scheduling. Considering the travel time uncertainty, especially when the traversal time on a link overlaps peak hours, the fleet management system has to plan the schedule and adapt to stochastic travel time. The components of the transport cost are not independent of each other. For example, the trade-off between the penalty on schedule miss and fuel cost will make a great difference if it is more profitable to form platoon and thus save fuel at the risk of incurring delay penalty. Potentially, the problem can be more complex if route change leads to fuel cost saving by platooning with other vehicles and compensate the unnecessary loss in travel time. However, we focus on the optimization on simple network, therefore departure time and platooning are the decisional variables being considered.

## 2.2 Fuel cost

For network at link resolution, both average-speed-based model and dynamic model can be used to calculate individual vehicle fuel consumption. In our case, the main difference of fuel consumption between follower vehicle in the platoon and not platooned vehicle lies in the external air drag force reduction, from the perspective of energy conservation. To take the effect of air drag into consideration, we refer to the model proposed in [6]. This model is an instantaneous model, which the fuel rate $FR$ is estimated as

$$FR = \frac{\xi}{\kappa\eta}\left(kN_eV + \frac{0.5C_d\rho Av^3(1-\varphi) + Mv(g\sin\alpha + gC_r\cos\alpha)}{1000\varepsilon\varpi}\right) \tag{2}$$

where $\xi$ is fuel-to-air mass ratio, $\kappa$ is the heating value of the fuel, $\eta$ is a conversion factor from grams to liters, $k$ is the engine friction factor, $N_e$ is the engine speed, $V$ is the engine displacement, $\rho$ is the air density, $A$ is the vehicle front area, $v$ is speed, $M$ is the total vehicle weight, $g$ is the gravitational constant, $\alpha$ is the road gradient, $C_d$ and $C_r$ are the coefficient of aerodynamic drag and rolling resistance, $\varepsilon$ is vehicle drive train efficiency, $\varpi$ is an efficiency parameter for engine and $\varphi$ is the air-drag reduction factor. If the vehicle is a follower in the platoon, $\varphi = 0.1$, otherwise $\varphi = 0$. In real traffic, vehicle speed $v$ will not be a constant during the long haul, so the speed profile will change from time to time. Theoretically this instantaneous model is more applicable if the second-by-second speed is used and the total fuel consumption is calculated as the accumulation of the fuel cost in each time step. For the purpose of simplification, average speed is used to replace the instantaneous speed in our study and the road gradient $\alpha$ is assumed to be 0. When traversing a distance of $d$, multiplying the fuel rate $FR$ by the travel time $T = d/v$, the total amount of fuel used is obtained:

$$F = B_1\left(B_2T + B_3\frac{d^3}{T^2} + B_4d\right) \tag{3a}$$

$$B_1 = \frac{\xi}{\kappa\eta}, B_2 = kN_eV, B_3 = \frac{0.5C_d\rho A(1-\varphi)}{1000\varepsilon\varpi}, B_4 = \frac{MgC_r}{1000\varepsilon\varpi} \tag{3b}$$

As indicated by the model, fuel consumption consists of three ingredients: the energy needed to overcome the frictional resistance and air drag and the basic consumption to maintain the operation of the cylinder. Since the explicit introduction of the air-drag effect, the fuel consumption model becomes a nonlinear function of travel time, reflecting that driving at a rather high speed (very short average travel time) and encountering heavy traffic congestion (very long average travel time) will both lead to drastic increase in fuel consumption. Under this condition, the fuel saving by platooning of two vehicles will be

$$F_s = \frac{\xi}{\kappa\eta} \cdot \frac{0.05C_d\rho A}{1000\varepsilon\varpi} \cdot \frac{d^3}{T^2} \tag{4}$$

Note that $d$ here is the common path of the two vehicles and we assume these vehicles have identical vehicle parameters. Given other parameters are constant, $F_s \propto \frac{d^3}{T^2}$. This shows the main approach to increase the fuel saving by platooning is to extend the common path of these vehicles as much as possible. The coordinated route optimization by [10] in Chapter 5 belongs to this sort, where the author seeks for the fuel-optimal path rather than the shortest path. In terms of reducing travel time $T$, which will also change the total fuel consumption, may lead to opposite outcome regarding fuel cost, as illustrated by Equation 3.

Table 1: Notation of model parameters

| Notation | Description |
|---|---|
| $\mathcal{X}$ | Head start, decision variable |
| $\tau$ | Delay, random variable |
| $T_c$ | Travel time, constant |
| $t_d$ | Departure time |
| $t_a$ | Preferred arrival time |
| $\delta$ | Preferred arrival time difference, $\delta = t_{a2} - t_{a1} > 0$ |
| $F$ | Fuel consumption |
| $\phi(\tau)$ | Probability density function of $\tau$ |
| $\Phi(\tau)$ | Cumulative distribution function of $\tau$ |
| $w_t$ | Unit cost of travel time |
| $w_e$ | Unit cost of early penalty |
| $w_l$ | Unit cost of late penalty, $w_e < w_l$ |
| $w_f$ | Unit cost of fuel consumption |

## 2.3 Driving cost and schedule miss penalty

The driving cost item can be interpreted as the driver wage, which is evaluated by total travel time and unit cost of travel time $w_t$. It is worthwhile to mention that travel time is sensitive to state changes in the transport system. Factors that affect humans, vehicles and facilities, which are the main components of the system, will contribute to travel time fluctuations. In particular, different vehicles traversing the same link during the same time interval may experience different travel times. Respecting the fact that travel time is not stationary in real traffic, the total travel time is modelled as the deterministic part $T_c$ plus the uncertain part $\tau$. By the constant $T_c$, which can be acquired from historical data, the lower bound of the total travel time is decided. The uncertain part $\tau$ is introduced to describe the non-recurrent delay. In this study, $\tau$ is assumed to be a non-negative random variable.

Since $\tau$ is unknown at the time when the decision departure time should be made, the realization of $\tau$ will trigger the early arrival or late arrival event. Some authors, including Robert & Kenneth [12], use the terms "schedule delay early" and "schedule delay late" to name these two situations. If the actual arrival time is earlier than the preferred arrival time $t_a$, an early penalty is incurred. Otherwise a late penalty is incurred. Here early penalty and late penalty are collectively referred to as the schedule miss penalty.

# 3 Common route problem

## 3.1 Optimal decision for single HDV

For the single-vehicle case, the cost function comprises four parts: travel cost, penalty for early arrival, penalty for late arrival and fuel consumption. The cost function of a single vehicle has the form [1]:

$$\mathbf{G}(t_d, \tau) = w_t(T_c + \tau) + w_e[t_a - (t_d + T_c + \tau)]_+ + w_l[t_d + T_c + \tau - t_a]_+ + w_f F \tag{5}$$

Let $\mathcal{X} = t_a - t_d - T_c$, where $\mathcal{X}$ can be taken as the time interval reserved for non-recurrent delay, or "head start". If the actual delay turns out to be greater than $\mathcal{X}$, then a late penalty is incurred. Equation 5 can be rewritten as:

$$\mathbf{H}(\mathcal{X}, \tau) = w_t(T_c + \tau) + w_e[\mathcal{X} - \tau]_+ + w_l[\tau - \mathcal{X}]_+ + w_f F \tag{6}$$

Here $\mathcal{X}$ is the decision variable and $\tau$ is a parameter. Since $\mathcal{X}$ only appears in the penalty items, and $\tau$ is a random variable, in essence the optimization problem is transformed correspondingly into

$$\min_{\mathcal{X} \geq 0}\{h(\mathcal{X}) := \mathbb{E}[w_e[\mathcal{X} - \tau]_+ + w_l[\tau - \mathcal{X}]_+]\} \tag{7}$$

where the minimum is attained when $\bar{\mathcal{X}} = \tau$. For a single vehicle, arrival on time will give the smallest penalty, and thus the smallest cost, since the driver wage and fuel consumption part are both independent of departure time $t_d$. For $\mathcal{X} \geq 0$,

$$h(\mathcal{X}) = h(0) + \int_0^{\mathcal{X}} h'(z)dz \tag{8}$$

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[1]For a number $a \in \mathbb{R}$, $[a]_+$ denotes the maximum $\max\{a, 0\}$.

where at non-differentiable points the derivative $h^{'}(z)$ is understood as the right hand side derivative. Since $\tau \geq 0$,

$$h(0) = w_l\mathbb{E}[\tau] \tag{9}$$

$$
\begin{aligned}
h^{'}(z) &= \frac{d}{dz}\mathbb{E}[w_e\max\{z - \tau, 0\}] + \frac{d}{dz}\mathbb{E}[w_l\max\{\tau - z, 0\}] \\
&= w_e\text{Prob}(\tau \leq z) + w_l(-\text{Prob}(\tau \geq z)) \\
&= w_e\Phi(z) - w_l(1 - \Phi(z)) \\
&= -w_l + (w_e + w_l)\Phi(z)
\end{aligned}
\tag{10}
$$

where $\Phi(\cdot)$ is cumulative distribution function. Thus

$$h(\mathcal{X}) = w_l\mathbb{E}[\tau] - w_l\mathcal{X} + (w_e + w_l)\int_0^{\mathcal{X}}\Phi(z)dz \tag{11}$$

Take the derivative of $h(\mathcal{X})$ with respect to $\mathcal{X}$ and set it equal to $0$, the optimal $\mathcal{X}$ is obtained as follow provided that $\Phi(\cdot)$ is continuous at $\mathcal{X}$:

$$\Phi(\mathcal{X}^*) = \frac{w_l}{w_e + w_l} \tag{12}$$

For distributions such as uniform distribution and exponential distribution, the optimal solution $\mathcal{X}^*$ is unique. Let

$$\int_0^x \Phi(z)dz = \mathcal{J}(x) \tag{13}$$

Given the optimum $\mathcal{X}^*$, the minimum expected penalty and the minimum expected cost can be calculated by substituting Equation 12 into Equation 11 and Equation 6 respectively:

$$h(\mathcal{X}^*) = w_l\mathbb{E}[\tau] - w_l\mathcal{X}^* + (w_e + w_l)\mathcal{J}(\mathcal{X}^*) \tag{14}$$

$$\mathbb{E}[\mathbf{H}(\mathcal{X}^*, \tau)] = w_t T_c + (w_t + w_l)\mathbb{E}[\tau] - w_l\mathcal{X}^* + (w_e + w_l)\mathcal{J}(\mathcal{X}^*) + w_f\mathbb{E}[F] \tag{15}$$

## 3.2  Two vehicles with platooning opportunity

For two vehicles of which the preferred arrival times are $t_{a1}$ and $t_{a2}$ respectively, if $t_{a1} = t_{a2}$, forming a platoon will give a more beneficial result. However, if $t_{a1} \neq t_{a2}$, forming a platoon by departing at the same time will cause one or two vehicles to experience more schedule penalty. The trade-off lies in whether the schedule penalty can be compensated by the weighted fuel saving $w_f F_s$. The cost function can be written in the following form:

$$
\begin{aligned}
\mathbf{G}_p(t_d, \tau) = \ & 2w_t(T_c + \tau) + w_e[t_{a1} - (t_d + T_c + \tau)]_+ + w_l[t_d + T_c + \tau - t_{a1}]_+ \\
& + w_e[t_{a2} - (t_d + T_c + \tau)]_+ + w_l[t_d + T_c + \tau - t_{a2}]_+ + 2w_f F - w_f F_s.
\end{aligned}
\tag{16}
$$

Now assume

$$
\begin{aligned}
\mathcal{X}_p &= t_{a1} - t_d - T_c \tag{17} \\
\delta &= t_{a2} - t_{a1} \tag{18}
\end{aligned}
$$

then $t_{a2} - t_d - T_c = \mathcal{X}_p + \delta$. Assume $t_{a2} > t_{a1}$, i.e. $\delta > 0$. The equivalent of Equation 7 is the following optimization problem:

$$\min_{\mathcal{X}_p \geq -\delta}\{h_p(\mathcal{X}_p) := \mathbb{E}[w_e[\mathcal{X}_p - \tau]_+ + w_l[\tau - \mathcal{X}_p]_+ + w_e[\mathcal{X}_p + \delta - \tau]_+ + w_l[\tau - (\mathcal{X}_p + \delta)]_+]\} \tag{19}$$

$-\delta$ is the lower bound of the possible optimal solution to this problem. Since both vehicles' late arrival or early arrival will definitely not end up with the minimum cost, $\mathcal{X}_p \geq -\delta$ guarantees both vehicles will not arrive with $\text{Prob}(late) = 1$.

Similarly, we can get

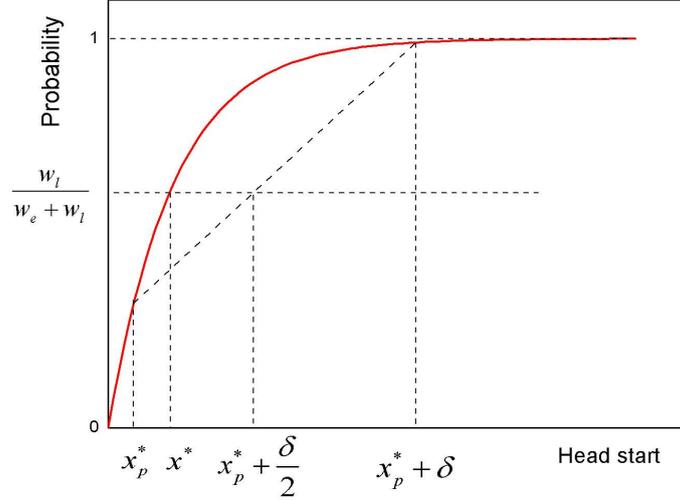$$h_p(\mathcal{X}_p) = h_p(-\delta) + \int_{-\delta}^{\mathcal{X}_p} h_p^{'}(z)dz \tag{20}$$

Figure 1: The relationship between $\mathcal{X}_p^*$, $\mathcal{X}_p^* + \delta$ and $\mathcal{X}^*$

$$h_p(-\delta) = 2w_l\mathbb{E}[\tau] + w_l\delta \tag{21}$$

$$
\begin{aligned}
h_p'(z) &= w_e\Phi(z) - w_l(1 - \Phi(z)) + w_e\Phi(z + \delta) - w_l(1 - \Phi(z + \delta)) \\
&= (w_e + w_l)(\Phi(z) + \Phi(z + \delta)) - 2w_l
\end{aligned} \tag{22}
$$

Since $-\delta$ is a constant and $h(-\delta)$ is independent of $\mathcal{X}_p$, take the derivative of the right-hand side of Equation 20 with respect to $\mathcal{X}_p$ and equate it to $0$ will give the result:

$$\frac{\Phi(\mathcal{X}_p^*) + \Phi(\mathcal{X}_p^* + \delta)}{2} = \frac{w_l}{w_e + w_l} \tag{23}$$

Compare Equation 23 with Equation 12 and we will find $\Phi(\mathcal{X}^*)$ equals the average of $\Phi(\mathcal{X}_p^*)$ and $\Phi(\mathcal{X}_p^* + \delta)$, which can be illustrated by Figure 1.

the minimum expected cost of penalty is obtained (notice that if $z < 0, \Phi(z) = 0$ and $\mathcal{J}(0) = 0$ according to Equation 13):

$$h_p(\mathcal{X}_p^*) = -w_l\delta + 2w_l\mathbb{E}[\tau] - 2w_l\mathcal{X}_p^* + (w_e + w_l)[\mathcal{J}(\mathcal{X}_p^* + \delta) + \mathcal{J}(\mathcal{X}_p^*)] \tag{24}$$

Compared with the situation that two vehicles depart at their respective optimal departure time, the loss of two platooned vehicles, denoted by $Q$, is the penalty on schedule miss and the gain is the weighted fuel saving $w_f F_s$. According to Equation 14 and Equation 24, the loss can be expressed as follows:

$$
\begin{aligned}
Q =&\, h_p(\mathcal{X}_p^*) - 2h(\mathcal{X}^*) \\
=&\, -w_l\delta - 2w_l\mathcal{X}_p^* + 2w_l\mathcal{X}^* + (w_e + w_l)[\mathcal{J}(\mathcal{X}_p^* + \delta) + \mathcal{J}(\mathcal{X}_p^*) - 2\mathcal{J}(\mathcal{X}^*)]
\end{aligned} \tag{25}
$$

If $\delta = 0$, i.e. two vehicles are supposed to arrive at the same time, $\mathcal{X}_p^*$ will be equal to $\mathcal{X}^*$ and the loss will be 0. If the loss is smaller than the gain, i.e. $Q < \mathbb{E}[w_f F_s]$, two vehicles should form platoon. Otherwise, two vehicles should drive individually.

## 3.3 Numerical example

In order to analyze the optimal head start and platooning opportunity with respect to preferred arrival time difference $\delta$ and travel time variance, we use a set of unit cost coefficients. Following [6], we estimate the driver wage by $w_t = 0.1877$ €/min (converted from $0.0022$ £/s). As for $w_e$ and $w_l$, they are much dependent on the sort of goods the vehicle delivers, varying from machine components, chemicals to milk. In our study, we assume $w_e = 0.1$ €/min, $w_l = 1.0$ €/min, which means being late is much more serious than being early. Moreover, $w_f = 1.5$ €/liter [1]. Other parameters involved are: $\xi = 1$,
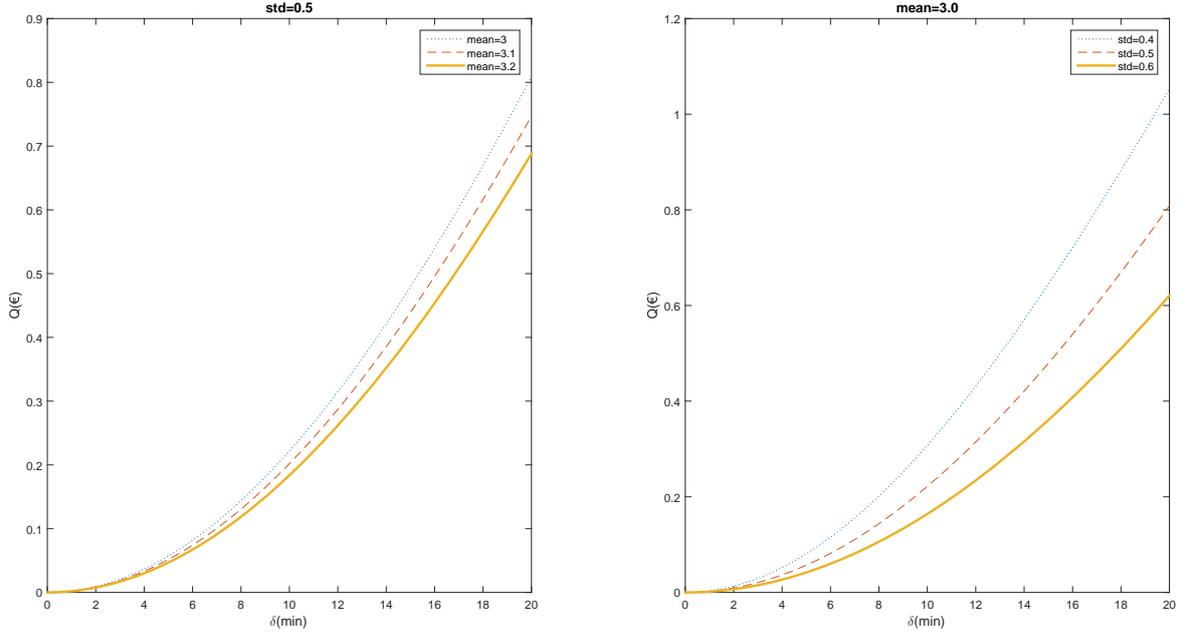
Figure 2: The relationship between $\delta$ and $Q$

$\kappa = 44$ kJ/g, $\eta = 737$ g/l, $k = 0.2$ kJ/rev/l, $N_e = 33$ rev/s, $V = 5$ l, $\rho = 1.2041$ kg/m$^3$, $A = 3.912$ m$^2$, $g = 9.81$ m/s$^2$, $C_d = 0.7$, $C_r = 0.01$, $\varepsilon = 0.4$, $\varpi = 0.9$ [6] and $M = 20000$ kg. The common route length is 100km and the deterministic part of travel time is $T_c = 46$min.

### 3.3.1   The impact of preferred arrival time difference $\delta$

To analyze the impact of preferred arrival time difference $\delta$ on the loss $Q$, the relationship between $\mathcal{X}_p^*$ and $\delta$ should be considered. If $\mathcal{X}_p^*$ is viewed as the implicit function of $\delta$ by Equation 23, where

$$P = \Phi(\mathcal{X}_p^*) + \Phi(\mathcal{X}_p^* + \delta) - \frac{2w_l}{w_e + w_l} \tag{26}$$

the derivative of $\mathcal{X}_p^*$ is given by

$$\begin{aligned} \frac{d\mathcal{X}_p^*}{d\delta} &= -\frac{\partial P/\partial \delta}{\partial P/\partial \mathcal{X}_p^*} \\ &= -\frac{\phi(\mathcal{X}_p^* + \delta)}{\phi(\mathcal{X}_p^*) + \phi(\mathcal{X}_p^* + \delta)} \end{aligned} \tag{27}$$

Since $\frac{d\mathcal{X}_p^*}{d\delta} < 0$, the result shows the optimal head start decreases as preferred arrival time difference increases. By substituting into Equation 23 and Equation 27, the derivative of $Q$ can be calculated and simplified:

$$\frac{dQ}{d\delta} = w_l - (w_e + w_l)\Phi(\mathcal{X}_p^*) \tag{28}$$

From Equation 23 we know that $\Phi(\mathcal{X}_p^*) < \frac{w_l}{w_e + w_l}$ and $\Phi(\mathcal{X}_p^* + \delta) > \frac{w_l}{w_e + w_l}$, thus $\frac{dQ}{d\delta}$ is always positive, i.e., as $\delta$ increases, platooning will cause more penalty loss, which requires more fuel saving to gain benefit. Figure 2 shows the impact of $\delta$ on the loss. Figure 3 shows the impact of $\delta$ on total cost, where the total cost after platooning is represented by solid lines and before platooning dashed lines.

### 3.3.2   The impact of mean and standard deviation of travel time

To study this problem, we can rewrite the stochastic delay $\tau$ in the form $\tau = \mu + \sigma\tau_0$, where $\tau_0$ is a normalized stochastic variable with mean 0, standard deviation 1, probability density function $\psi$ and cumulative distribution function $\Psi$. Then
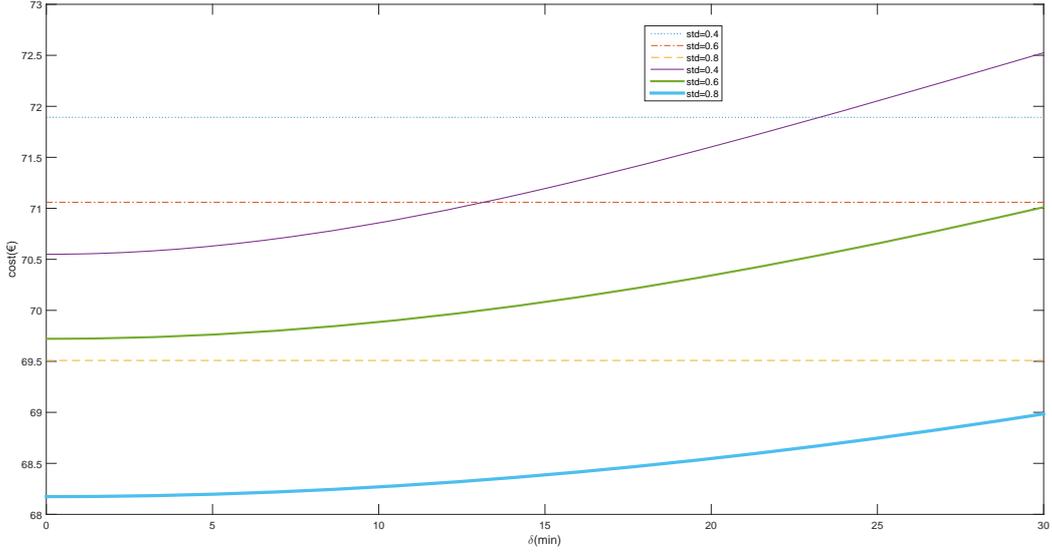
Figure 3: The relationship between $\delta$ and total cost when mean=3.0

$$h(\mathcal{X}) = w_e \int_{-\infty}^{\frac{\mathcal{X}-\mu}{\sigma}} (\mathcal{X} - \mu - \sigma\tau_0)\psi(\tau_0)d\tau_0 + w_l \int_{\frac{\mathcal{X}-\mu}{\sigma}}^{+\infty} (\mu + \sigma\tau_0 - \mathcal{X})\psi(\tau_0)d\tau_0 \tag{29}$$

The optimal head start $\mathcal{X}^*$ is obtained by solving $\frac{dh(\mathcal{X})}{d\mathcal{X}} = 0$:

$$\mathcal{X}^* = \sigma\Psi^{-1}\left(\frac{w_l}{w_e + w_l}\right) + \mu \tag{30}$$

Substituting Equation 30 into Equation 29, we can get

$$h(\mathcal{X}^*) = w_l\sigma \int_{\frac{w_l}{w_e+w_l}}^{1} \Psi^{-1}(u)du - w_e\sigma \int_{0}^{\frac{w_l}{w_e+w_l}} \Psi^{-1}(u)du \tag{31}$$

The result shows, for individually-driving vehicles, the optimal head start is linear in $\mu$ and $\sigma$. Besides, the minimum delay penalty cost is also linear in $\sigma$ but does not include the effect of $\mu$. This can be interpreted as the contribution of the chosen optimal head start, which will adjust to $\mu$ to guarantee $h(\mathcal{X}^*)$ not affected by the mean of delay.

For two platooned vehicles, the first-order condition is as follow:

$$\Psi\left(\frac{\mathcal{X}_p^* - \mu}{\sigma}\right) + \Psi\left(\frac{\mathcal{X}_p^* - \mu + \delta}{\sigma}\right) = \frac{2w_l}{w_e + w_l} \tag{32}$$

Unlike Equation 30, the result can't show the impact of $\mu$ and $\sigma$ explicitly. But it still gives some useful insight if we calculate the derivatives:

$$\frac{d\mathcal{X}_p^*}{d\mu} = 1 \tag{33}$$

$$\frac{d\mathcal{X}_p^*}{d\sigma} = \frac{\mathcal{X}_p^* - \mu}{\sigma} + \frac{\psi\left(\frac{\mathcal{X}_p^* - \mu + \delta}{\sigma}\right)\delta}{\sigma\left(\psi\left(\frac{\mathcal{X}_p^* - \mu}{\sigma}\right) + \psi\left(\frac{\mathcal{X}_p^* - \mu + \delta}{\sigma}\right)\right)} \tag{34}$$

$\mathcal{X}_p^*$ is linear in $\mu$ and non-linear in $\sigma$. The second part of Equation 34 is non-negative and for the first part, due to the fact that $\Psi$ is non-decreasing and no greater than 1, we can draw the conclusion that $\frac{\mathcal{X}_p^* - \mu}{\sigma} > 0$ from Equation 32 (note that $w_l > w_e$). Thus, Equation 34 is always positive, i.e., $\mathcal{X}_p^*$ will increase as $\sigma$ increases. Similarly,

$$h_p(\mathcal{X}_p^*) = -\delta w_l + (w_e + w_l)\delta\Psi\left(\frac{\mathcal{X}_p^* - \mu + \delta}{\sigma}\right) - (w_e + w_l)\sigma\left(\int_{0}^{\Psi\left(\frac{\mathcal{X}_p^* - \mu}{\sigma}\right)} \Psi^{-1}(u)du + \int_{0}^{\Psi\left(\frac{\mathcal{X}_p^* - \mu + \delta}{\sigma}\right)} \Psi^{-1}(u)du\right) \tag{35}$$
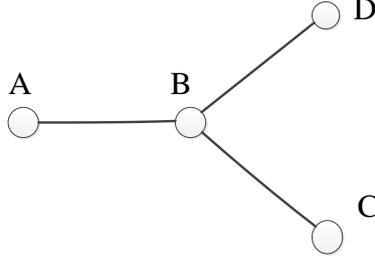
8

Figure 4: Network

When the delay penalty loss by platooning is exactly the same with the expected fuel saving, we can get the threshold of preferred arrival time difference $\delta_{max}$. $\delta_{max}$ satisfy

$$h_p(\mathcal{X}_p^*, \delta_{max}) - 2h(\mathcal{X}^*) - \int_{-\infty}^{+\infty} \frac{w_f B_1 B_3 d^3 \varphi}{(T_c + \mu + \sigma\tau_0)^2} \psi(\tau_0)d\tau_0 = 0 \tag{36}$$

$$\frac{d\delta_{max}}{d\mu} = \frac{M}{I} \tag{37}$$

$$\frac{d\delta_{max}}{d\sigma} = \frac{N}{I} \tag{38}$$

where

$$
\begin{aligned}
M &= -w_f B_1 B_3 d^3 \varphi \int_{-\infty}^{+\infty} \frac{2}{(T_c + \mu + \sigma\tau_0)^3} \psi(\tau_0)d\tau_0 \\
N &= (w_e + w_l)(\int_0^{\Psi(\frac{\mathcal{X}_p^* - \mu}{\sigma})} \Psi^{-1}(u)du + \int_0^{\Psi(\frac{\mathcal{X}_p^* - \mu + \delta_{max}}{\sigma})} \Psi^{-1}(u)du) + 2w_l \int_{\frac{w_l}{w_e + w_l}}^1 \Psi^{-1}(u)du \\
&\quad - 2w_e \int_0^{\frac{w_l}{w_e + w_l}} \Psi^{-1}(u)du - w_f B_1 B_3 d^3 \varphi \int_{-\infty}^{+\infty} \frac{2}{(T_c + \mu + \sigma\tau_0)^3} \tau_0 \psi(\tau_0)d\tau_0 \\
I &= w_l - (w_e + w_l)\Psi(\frac{\mathcal{X}_p^* - \mu}{\sigma})
\end{aligned}
\tag{39}
$$

# 4 Planning for a simple network

## 4.1 Optimal decision on platooning

Now consider the small network shown in Figure 4. Two vehicles travel along the routes $A \to B \to C$ and $A \to B \to D$, preferably forming platoon on link $l_{AB}$. Below we will investigate when it is possible to form platoon to pursue the minimized cost. Let $L_1$, $L_2$, $L_3$ denote the lengths of $l_{AB}$, $l_{BC}$, $l_{BD}$, and $T_{AB} + \tau_1$, $T_{BC} + \tau_2$, $T_{BD} + D_3$ the travel time on these links, where $T_{AB}$, $T_{BC}$ and $T_{BD}$ are constants and $\tau_1$, $\tau_2$ and $D_3$ are random variables with mutually independent probability density functions $f_1$, $f_2$ and $f_3$ respectively. For a single vehicle traveling along route $A \to B \to C$, the cost function can be written as

$$
\begin{aligned}
\mathbf{H}_{ABC}(\mathcal{X}_{ABC}, \tau_1, \tau_2) &= w_t(T_{AB} + T_{BC} + \tau_1 + \tau_2) + w_e[\mathcal{X}_{ABC} - (\tau_1 + \tau_2)]_+ \\
&\quad + w_l[(\tau_1 + \tau_2) - \mathcal{X}_{ABC}]_+ + w_f(F_{AB} + F_{BC})
\end{aligned}
\tag{40}
$$

where $\mathcal{X}_{ABC}$ is the head start and $F_{AB}$, $F_{BC}$ are fuel consumptions on $l_{AB}$, $l_{BC}$. Compared with Equation 6, Equation 40 differs mainly in random parameters and fuel cost, since both $T_{AB}$ and $T_{BC}$ are constants as $T_c$. Assume a new random variable $\tau_{1,2} = \tau_1 + \tau_2$ follows distribution $f_{1,2}$. Recall the convolution formula

$$f_{1,2}(\tau_{1,2}) = \int_{-\infty}^{+\infty} f_1(\tau_{1,2} - \tau_2) f_2(\tau_2) d\tau_2$$
$$= \int_{-\infty}^{+\infty} f_1(\tau_1) f_2(\tau_{1,2} - \tau_1) d\tau_1 \tag{41}$$

the first-order condition will have the same form as in Equation 12, only the inverse function changes to the distribution of $\tau_{1,2}$. Let $F_{1,2}$ denote the CDF of $\tau_{1,2}$ and $F_{1,3}$ the CDF of $\tau_{1,3}(\tau_{1,3} = \tau_1 + \tau_3)$, the respective optimal departure time are:

$$\mathcal{X}_1^* = F_{1,2}^{-1}\left(\frac{w_l}{w_e + w_l}\right) \tag{42}$$

$$\mathcal{X}_2^* = F_{1,3}^{-1}\left(\frac{w_l}{w_e + w_l}\right) \tag{43}$$

System optimum penalty cost function:

$$\min_{\mathcal{X}_p \geq -\delta} \{h_p(\mathcal{X}_p) := \mathbb{E}[w_e[\mathcal{X}_p - \tau_{1,2}]_+ + w_l[\tau_{1,2} - \mathcal{X}_p]_+ + w_e[\mathcal{X}_p + \delta - \tau_{1,3}]_+ + w_l[\tau_{1,3} - (\mathcal{X}_p + \delta)]_+]\} \tag{44}$$

where

$$\mathcal{X}_p = t_{a1} - t_d - T_{AB} - T_{BC} \tag{45}$$
$$\delta = t_{a2} - t_{a1} + T_{BC} - T_{BD} \tag{46}$$

Assume $\delta > 0$, and the first-order condition leads to

$$\frac{F_{1,2}(\mathcal{X}_p^*) + F_{1,3}(\mathcal{X}_p^* + \delta)}{2} = \frac{w_l}{w_e + w_l} \tag{47}$$

$$h_p(\mathcal{X}_p^*) = -w_l\delta + w_l\mathbb{E}[\tau_{1,2} + \tau_{1,3}] - 2w_l\mathcal{X}_p^* + (w_e + w_l)[J_{1,3}(\mathcal{X}_p^* + \delta) + J_{1,2}(\mathcal{X}_p^*)] \tag{48}$$

$$h(\mathcal{X}_1^*) = w_l\mathbb{E}[\tau_{1,2}] - w_l\mathcal{X}_1^* + (w_e + w_l)J_{1,2}(\mathcal{X}_1^*) \tag{49}$$

$$h(\mathcal{X}_2^*) = w_l\mathbb{E}[\tau_{1,3}] - w_l\mathcal{X}_2^* + (w_e + w_l)J_{1,3}(\mathcal{X}_2^*) \tag{50}$$

$$Q = -w_l\delta + w_l(\mathcal{X}_1^* + \mathcal{X}_2^* - 2\mathcal{X}_p^*) + (w_e + w_l)[J_{1,3}(\mathcal{X}_p^* + \delta) + J_{1,2}(\mathcal{X}_p^*) - J_{1,2}(\mathcal{X}_1^*) - J_{1,3}(\mathcal{X}_2^*)] \tag{51}$$

If $Q < \mathbb{E}[w_f F_s]$, two vehicles should form platoon.

## 4.2 Case study

Table 2 shows the effects of different parameters which influence the decision variable $\mathcal{X}_p^*$ and total cost after platooning for the simple network problem with $t_{a2} - t_{a1} = 10$min. Knowing the tendency of the decision variable changing according to the proceeding sections, some typical instances are picked out simply by changing one parameter in each instance. The main changes in the simple network problem compared with the common route problem are: i) the different $\delta$; ii) the involvement of different travel lengths; iii) the joint distribution. The joint distribution makes the prediction based on mean and variance of the delay more complicated, but still the model and the first-order condition have the same form as that in section 3, only the distribution function changes. However, the meaning of $\delta$ in this part is different since the introduction of $T_{BC} - T_{BD}$ and will no longer be the original preferred arrival time difference. In addition, in section 3 we assume $\delta > 0$ and not much difference will be made if we change the order of $t_{a1}$ and $t_{a2}$ to make sure $\delta > 0$ always holds. But here the absolute value of $t_{a2} - t_{a1}$ does not make final decision. Instances 1,2 and 3 show, as $T_{BD}$ increases, $\mathcal{X}_p^*$ first increases and then decreases, as well as the according penalty loss $Q$. Instances 4,5,6 and 7 are to show the impacts of travel time variances. As mentioned in section 3, either greater mean or greater standard deviation of delay will lead to a bigger head start, i.e., departing earlier. In particular, the weighted fuel saving and cost are very remarkable in instance 5, due to the increased $\sigma 1$ which indirectly increases the expected fuel saving as we tell from the fuel saving model. Also, instance 5 is a typical example illustrating that increased fuel saving will not necessarily lead to increased total cost. Instances 8,9 and 10 are targeted to show the impact of the route lengths. Since the $\mathcal{X}_p^*$ is independent of the route lengths, the schedule loss will stay unchanged.

Table 2: Summarized results for 10 sets of instances regarding different parameters

| No. | $T_{AB}$ | $T_{BC}$ | $T_{BD}$ | $\mu_1$ | $\mu_2$ | $\mu_3$ | $\sigma_1$ | $\sigma_2$ | $\sigma_3$ | $L_1$ | $L_2$ | $L_3$ | $\mathcal{X}_p^*$ | $Q$ | $w_f F_s$ | Cost |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 5 | 1 | 6 | 3 | 3.1 | 3.1 | 0.4 | 0.5 | 0.5 | 70000 | 50000 | 60000 | 66.817 | 0.0473 | 3.7881 | 187.3247 |
| 2 | 5 | 1 | 11 | 3 | 3.1 | 3.1 | 0.4 | 0.5 | 0.5 | 70000 | 50000 | 60000 | 69.09 | 0 | 3.7881 | 182.4398 |
| 3 | 5 | 1 | 16 | 3 | 3.1 | 3.1 | 0.4 | 0.5 | 0.5 | 70000 | 50000 | 60000 | 66.817 | 0.0473 | 3.7881 | 180.4859 |
| 4 | 5 | 1 | 6 | 3.4 | 3.1 | 3.1 | 0.4 | 0.5 | 0.5 | 70000 | 50000 | 60000 | 81.64 | 0.0411 | 2.0218 | 160.5562 |
| 5 | 5 | 1 | 6 | 3 | 3.1 | 3.1 | 0.6 | 0.5 | 0.5 | 70000 | 50000 | 60000 | 75.105 | 0.0354 | 4.5133 | 202.2585 |
| 6 | 5 | 1 | 6 | 3 | 3.4 | 3.1 | 0.4 | 0.5 | 0.5 | 70000 | 50000 | 60000 | 75.456 | 0.5889 | 3.7881 | 181.5002 |
| 7 | 5 | 1 | 6 | 3 | 3.1 | 3.1 | 0.4 | 0.6 | 0.5 | 70000 | 50000 | 60000 | 69.554 | 0.1807 | 3.7881 | 192.1209 |
| 8 | 5 | 1 | 6 | 3 | 3.1 | 3.1 | 0.4 | 0.5 | 0.5 | 80000 | 50000 | 60000 | 66.817 | 0.0473 | 5.6545 | 224.0959 |
| 9 | 5 | 1 | 6 | 3 | 3.1 | 3.1 | 0.4 | 0.5 | 0.5 | 70000 | 80000 | 60000 | 66.817 | 0.0473 | 3.7881 | 253.5063 |
| 10 | 5 | 1 | 6 | 3 | 3.1 | 3.1 | 0.4 | 0.5 | 0.5 | 7000 | 50000 | 80000 | 66.817 | 0.0473 | 3.7881 | 217.3559 |

# 5 Conclusions

We have introduced and analyzed a platoon scheduling problem considering travel time variance, which aims at minimize the total cost during the delivery. The model we propose explicitly accounts for the driving cost, schedule miss penalty and fuel cost, among which the attempts to arrive on time and save fuel by platooning are conflicting goals. We begin with a common route problem and study the impacts of travel time variance and preferred arrival time difference on the platooning opportunity and total cost without loss of generality. Then we extend the problem to a small network and study the influences of the route length and different deterministic parts of travel time. As a startup of the platoon scheduling problem, this model gives some useful insights regarding platooning decisions under uncertain circumstances, though the fuel saving is not remarkable due to limited vehicle amount and simple network. Further analysis will be carried out about platooning in a more complex network, but with the growth of the network, it is necessary to resort to simulation-based approach for decision support.

# Acknowledgments

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