2013 IEEE Intelligent Vehicles Symposium (IV)
June 23-26, 2013, Gold Coast, Australia

Optimal Controls of Fleet Trajectories for Fuel and Emissions

Xiaoliang Ma*

***Abstract*—Increased demand for transport, coupled with energy, climate and environmental concerns, has put more and more pressure for improved performance on traffic systems. The recent development in vehicle-to-infrastructure (V2I) communication provides an effective means for continuous management of vehicle driving. This study presents an essential step of the work towards a dynamic fleet management system that takes advantages of real-time traffic information and communication. Based on the optimal control theory, a methodological approach is developed to control the environmental impacts of live vehicle fleets. In particular, vehicle trajectories that minimize local environmental objectives are derived by applying a discrete dynamic programming method. Numerical examples show that the method is promising for local V2I based traffic management applications and can be further extended for more complex optimal control problems in dynamic fleet management.**

I. INTRODUCTION

Closely coupled with the growing economy of the world is the steadily increasing demand for transport, and corresponding rapid augmentation in energy consumption and green house gases produced. Transportation systems has contributed to a major part of the increase in oil consumption during the last decades and the growth is expected to continue. Its development therefore faces big challenges with respect to the social sustainability. The environmental footprint of transport in the European Union (EU) corresponds to almost 25% of the total greenhouse gas emissions and 30% of the CO_2. Hence, the entire transport sector, and particularly road transport, has been targeted as a main policy area where further environmental and overall efficiency improvements are critical for a sustainable future of European transportation.

A. Background

While the demand for road transport has been skyrocketing during the last decade, the impacts on air pollution in cities, global climate change and other environmental aspects need to be significantly reduced and road congestion better controlled to satisfy social requirement. To ensure sustainability and global acceptance of commercial transportation, new systems which reduce the dependence on oil and minimize emission of greenhouse gases need to be developed. One of the goals for 2030, set by the European Commission, is to reduce the green house gas emissions to 80% of the 2008 level [1], and at the same time, the goods transport in Europe is projected to increase by 75% [2].

The rapid evolution of information and communication technology (ICT) presents an excellent opportunity to tackle these problems through novel integrated intelligent transportation system (ITS) solutions. Recent advances in application of communication technology have opened up new perspectives for the development of ITS systems that utilize vehicle-to-vehicle (V2V) and/or vehicle-to-infrastructure (V2I) communication, so called cooperative systems. The expectation of the industry and other stakeholders is that cooperative systems may improve safety, efficiency and reduce the environmental impacts of road traffic in an even more efficient way than the existing ITS systems in operation.

Platooning and fleet management are an active research area that has attracted strong interests from transport stakeholders, especially producers and owners of freight fleets. Whereas much research has been dedicated to vehicle and control technologies that allow vehicle platoons to operate on normal public highways with significant environmental, safety and comfort benefits, it is also important to integrate platooning operation with real-time traffic information and vehicle states. The recent FP7 projects, SATRE[1] and HaveIT[2], reflect the trends of these technologies. It was even shown in previous tests that platooning application can save 5% to 15% fuel usage and significantly reduce carbon footprints of heavy trucks due to less aerodynamic drag [3], [4]. In Sweden, major fleet producers/owners such as Scania have formulated joint research effort with research institutions e.g. KTH in developing control strategies for platooning operation as well as advanced in-vehicle technologies [4], [5], [6].

B. Research objectives

The application of communication technology in ITS development has attracted broad attention within EU. Many completed and ongoing EU projects have contributed to the state-of-the-art of cooperative systems while more attentions are given to vehicle adhoc network and V2V based ITS systems to improve road safety e.g. COOPERS [7]. Nevertheless, few infrastructure-based cooperative systems, according to our knowledge, have been developed to manage traffic fleets on road for energy and environmental purposes. There are increasing demand to develop intelligent infrastructure or roadside units that can achieve local management purpose according to live traffic conditions.

Since the fall of 2011, KTH and Scania AB have launched a joint research project, iQFleet, with an objective to develop intelligent real-time fleet management systems. A proposed system architecture is shown in figure 1, consisting of three layers. The first layer includes on-board sensors such as radar

Xiaoliang Ma Ph.D. is with the ITS Lab, Traffic and Logistics, KTH Royal Institute of Technology, Teknikringen 72, 10044, Stockholm, Sweden.
* Corresponding author, phone: +46 8 7908426, fax: +46 8 7908461, e-mail: liang@kth.se

[1] http://sartre-project.eu/
[2] http://www.haveit-eu.org/

and lidar, collecting information of nearby traffic and environment. Whenever traffic condition changes, either driver or vehicle automatically makes a corresponding reaction. Adaptive cruise control (ACC) and active braking systems (ABS) are examples of such kind. In fleet management, these systems can be integrated to enable platooning, where multiple heavy-duty vehicles may drive close to each other so that air drag and hence energy consumption are both reduced. The second layer incorporates GPS positioning and wireless communication between vehicles. Therefore, information can be propagated beforehand, and vehicles can provide faster reaction to disturbance and emergency situations in traffic flow. The two layers above support vehicle fleets to operate as a whole unit and make joint decisions. From the fleet management point of view, a platoon can be formulated dynamically and interact with other platoons and traffic objects.

In the last layer, local traffic information can be transmitted to a central processing node able of creating local and regional view of traffic flows. Traffic models are then enabled to predict real-time dynamic traffic states, which in return be applied for on-line guidance to vehicles and fleets. Conventionally, such guidance is mainly through road-side messages or signs e.g. speed limits and other warnings. Due to the availability of vehicle infrastructure communication, it is possible to combine such guidance with in-vehicle systems and develop new traffic management measures at the individual vehicle or fleet level e.g. speed management. Drivers therefore may receive more frequent guidance that is based on dynamic traffic states.

This study is mainly focused on developing infrastructure-based measures that continuously manage vehicle and fleet states, in particular speed profiles, using communication channels. The basic idea of the paper is to derive local speed management strategies for vehicle and fleet corresponding to local traffic conditions and achieve minimal energy and environmental impacts. The paper focuses on the simpler case of a single vehicle, and it is natural to extend to the more complex case of fleets in the future study of the project. The next section derives an analytical approach to find the optimal speed control at the individual vehicle level when the initial and final states of the vehicle are known.

II. METHODS

Optimal control of automobiles for fuel economy is not a fully new research topic. Since 1980s, several pioneer studies [8] [9] have been conducted to investigate the underlining principle of optimal driving with an objective of minimizing fuel consumption. Detailed vehicle dynamics was modeled while terrain variation of highway was also considered. However, the approach has, unfortunately, not been applied in applications of traffic management and in-vehicle systems due to the limited availability of infrastructure, sensing technologies, and method and tool for traffic modeling and prediction at the time. The appearance of cooperative system in ITS development makes it technically possible to implement dynamic guidance to drivers with benefits not only in system efficiency but in fuel economy and environmental impacts. Thus, the eco-driving strategies become more than technical concepts. The recent ongoing EU projects such as WiSafeCar[3] and CoMoSeF[4] have shown the promising future of such application. In the iQFleet project, it is essential to obtain optimal controls for individual vehicles and fleets. Ideally, such type of controls can be disseminated to vehicles via infrastructure to vehicle (I2V) information and driver (or vehicle) will automatically follow the control messages received in real time. It is obvious that if the fuel usage and emission impacts can be minimized for each vehicle the controls will minimize the total emission and energy impacts of the traffic fleet.

The rest of the section starts by introducing the mathematical formulation and solution principle for a regular optimal control problem. The detailed vehicle trajectory control problem will then be treated using a discrete dynamic programming formulation in the context of cooperative traffic management.

A. Mathematical problem statement

General formulation*:* Consider a general optimal control problem that is described by the following ordinary differential system:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \boldsymbol{\alpha}(t)) \quad t \geq 0 \\ \mathbf{x}(0) = \mathbf{x}_0 \end{cases} \tag{1}$$

where $\mathbf{x}(t)$ is the state of the system controlled whereas $\boldsymbol{\alpha}(t)$ is the control input at time t. The initial boundary condition is given by $\mathbf{x}_0$. In a typical control problem, the target state $\mathbf{x}(\tau) = \mathbf{x}_1$ may be given and the control period τ is free. The control measure can be represented by a payoff function

$$J(\mathbf{x}, \boldsymbol{\alpha}) = \int_0^{\tau} r(\mathbf{x}(t), \boldsymbol{\alpha}(t))dt. \tag{2}$$

where $r(\mathbf{x}(t), \boldsymbol{\alpha}(t))$ is the general running payoff. The basic problem is to find an optimal control $\boldsymbol{\alpha}^*(\cdot)$ such that the control measure is optimized i.e.

$$J(\mathbf{x}^*, \boldsymbol{\alpha}^*) = \min_{\boldsymbol{\alpha} \in \Omega} J(\mathbf{x}, \boldsymbol{\alpha}). \tag{3}$$

where $\mathbf{x}^*$ is the optimal intermediate system states or trajectory.

This problem has been treated extensively in literature e.g. [10][11][12] and is often solved by reformulating optimal control as an optimization problem. That is, by defining the control theory Hamiltonian function $H(\mathbf{x}^*(t), \boldsymbol{\lambda}^*(t), \boldsymbol{\alpha}^*(t))$ as follows

$$H(\mathbf{x}(t), \boldsymbol{\lambda}(t), \boldsymbol{\alpha}(t)) = r(\mathbf{x}(t), \boldsymbol{\alpha}(t)) + \boldsymbol{\lambda}^T(t)\mathbf{f}(\mathbf{x}(t), \boldsymbol{\alpha}(t)) \tag{4}$$

[3] http://www.wisafecar.com/

[4] http://www.celtic-initiative.org/Projects/Celtic-Plus-Projects/2011/COMOSEF/comosef-default.asp

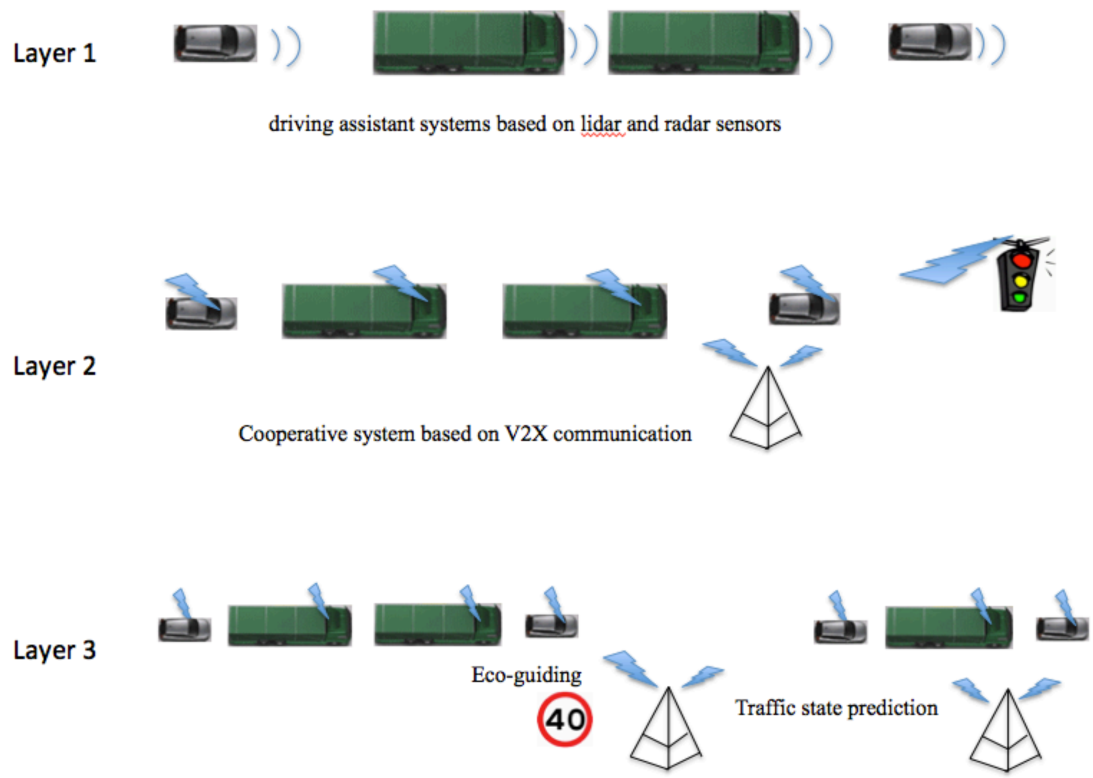

Figure 1. The concept framework of an intelligent fleet management system based on dynamic vehicle sensing and communications.

the Pontryagin Minimum Principle (PMP) is applied with the equations derived to satisfy the necessary condition to the infinite problem. $\boldsymbol{\lambda}(t)$ is a vector of adjoint variables at each time instant t, which are indeed Lagrange multipliers. It is shown that the optimal control problem formulated above can be solved by handling the following equations [11]:

$$H(\mathbf{x}^*(t), \boldsymbol{\lambda}^*(t), \boldsymbol{\alpha}^*(t)) = \min_{\boldsymbol{\alpha}\in\Omega} H(\mathbf{x}^*(t), \boldsymbol{\lambda}^*(t), \boldsymbol{\alpha}) \quad (5)$$

$$\dot{\mathbf{x}}^*(t) = \nabla_{\boldsymbol{\lambda}} H(\mathbf{x}^*(t), \boldsymbol{\lambda}^*(t), \boldsymbol{\alpha}^*(t)) \quad (6)$$

$$\dot{\boldsymbol{\lambda}}^*(t) = -\nabla_{\mathbf{x}} H(\mathbf{x}^*(t), \boldsymbol{\lambda}^*(t), \boldsymbol{\alpha}^*(t)) \quad (7)$$

This transfers the optimal control problem to point-wise optimization problems at each time instant. When boundary conditions are defined, analytical or numerical solutions may exist [10][12].

Control of vehicle trajectory: In the application of vehicle trajectory control, instantaneous vehicle state can be represented by a vector

$$\mathbf{x}(t) = [s(t)\ v(t)]^T \quad (8)$$

where $s(t)$ and $v(t)$ are vehicle traveling distance and speed at certain time t. The control input is the acceleration itself

$$\boldsymbol{\alpha}(t) = a(t). \quad (9)$$

This formulation neglects the detailed vehicle dynamics model in previous studies [8] since our application is focused on driving guidance at the tactical level. However, it is convenient to add the vehicle acceleration model in the application of detailed vehicle control [6].

The objective of the control is to minimize certain impact measure such as fuel consumption or emission represented by

$$J(\boldsymbol{\alpha}) = \int_0^{\tau} \phi(\mathbf{x}(t), a(t))dt \quad (10)$$

with constraints to the state variables and control inputs

$$\dot{\mathbf{x}}(t) = [v(t)\ a(t)]^T \quad (11)$$

$$\mathbf{x}(0) = \mathbf{x}_0 \quad (12)$$

$$\mathbf{x}(\tau) = \mathbf{x}_T \quad (13)$$

$$a(t) \in (\hat{d}_{max},\ \hat{a}_{max}(v)] \quad (14)$$

where $\phi(\mathbf{x}(t), a(t))$ is instantaneous emission or fuel usage; $\hat{d}_{max}$ is the maximum deceleration and $\hat{a}_{max}(v)$ is maximum acceleration that vehicle engine can reach at the speed level.

B. Solution method based on dynamic programming

Discrete formulation: The problem formulated by equation (10)-(14) is a two point boundary value problem (BVP) for a continuous ODE that needs to be solved numerically in applications. This paper, however, formulates the vehicle dynamics as a discrete system model as follows:

$$\mathbf{x}_{k+1} = \mathbf{f}(k, \mathbf{x}_k, \boldsymbol{\alpha}_k) = \mathbf{A} \cdot \mathbf{x}_k + a_k \mathbf{b} \quad (15)$$

where $\mathbf{x}_k = [s_k\ v_k]^T$ and a_k are the state of the vehicle and control input, the acceleration, at a time instant k

respectively. $\mathbf{b} = [\Delta t^2/2 \ \Delta t]^T$ is stationary in this problem and Δt is the time interval. $\mathbf{A}$ is also stationary, that is

$$\begin{pmatrix} 1 & \Delta t \\ 0 & 1 \end{pmatrix}.$$

The objective is to treat with the optimization below

$$\min_{\mathbf{X},\boldsymbol{\alpha}} \{\hat{\phi}(\mathbf{x}_N) + J(k, \mathbf{x}_k)\} \tag{16}$$

$$J(k, \mathbf{x}_k) = \sum_{k=0}^{N-1} \phi(k, \mathbf{x}_k, a_k)\Delta t \tag{17}$$

with constraints

$$\mathbf{x}_0 = [0 \ \hat{v}_0]^T \tag{18}$$

$$\mathbf{x}_N = [L \ \hat{v}_N]^T \tag{19}$$

$$a_k \in (\hat{d}_{max}, \ \hat{a}_{max}(v_k)] \tag{20}$$

where $\phi(k, \mathbf{x}_k, a_k)$ is the instantaneous emission or energy usage; $\hat{\phi}(\mathbf{x}_N)$ is the final reward function and the equations (18) and (19) show the initial and final conditions.

Dynamic programming model*:* The discrete problem above can be treated as a backward dynamic programming recursion i.e.

$$J(k, \mathbf{x}) = \min_{a \in \alpha(k,\mathbf{x})} \{\phi_k(\mathbf{x}, a)\Delta t + J(k+1, \mathbf{x}_{k+1})\}, \quad k = N-1, ..., 0 \tag{21}$$

According to the principle of optimality [11], there exists an optimal control solution with feedback control in each stage obtained by

$$a_k^* = \arg \min_{a \in \alpha(k,\mathbf{x})} \{\phi_k(\mathbf{x}, a)\Delta t + J(k+1, \mathbf{x}_{k+1})\}. \tag{22}$$

The dynamic programming solution above is formulated based on the temporal discretization whereas the state variable has to be discretized at the same time. The time interval for the optimal control is unknown, though it is possible to find an optimal value by testing different feasible times.

Since the total distance of the controlled area is known in the two point BVP problem, a discretization on space is considered instead. Space discretization indeed simplifies the state transition dynamics i.e.

$$\mathbf{x}_{n+1} = \mathbf{f}(n, \mathbf{x}_n, \boldsymbol{\alpha}_n) = \mathbf{x}_n + \boldsymbol{\alpha}_n \tag{23}$$

where $\mathbf{x}_n = v_n$ is the state variable and the control input is

$$\boldsymbol{\alpha}_n = \Delta v_n \tag{24}$$

$$a_n = [(v_n + \Delta v_n)^2 - v_n^2]/2\Delta s \tag{25}$$

$$\Delta t_n = \Delta v_n / a_n \tag{26}$$

Δt_n is the time interval and a_n is the acceleration. While the time intervals are not equalized, the dynamic programming model in this case is similar to that of the temporal discretization case i.e.

$$J(n, \mathbf{x}) = \min_{a} \{\phi_n(\mathbf{x}, a)\Delta t_n + J(n+1, \mathbf{x}_{n+1})\}, \quad n = N-1, ..., 0 \tag{27}$$

C. Fuel and emission estimation approach

In order to manage fuel consumption or other emission impacts induced by vehicle fleet, it is essential to accurately estimate the time-resolved emission quantity. The estimation is based on computational modeling of emissions using the inputs of traffic dynamics as well as individual vehicle information. Different emission models have been developed to compute fuel consumption and air pollutants including CO, HC, NOx and particulate matters (PMs) as well as green house gases e.g. CO_2.

Similar to traffic models, an emission model can be either macroscopic, mesoscopic or microscopic in nature according to its characteristics. In this study, detailed instantaneous emissions of vehicle fleet need to be estimated. Therefore, microscopic emission models [13] are most suitable for the application context. Taking vehicle operating conditions as inputs e.g. instantaneous speed, acceleration, engine states and so on, microscopic emission models aim at predicting vehicular emission at a second-by-second resolution. The development of such models normally requires collection of emission data from various types of vehicles. Different approaches have been adopted in the model development including the physical, power demand based [14] and the regression based approaches [15]. Whereas the power-based models are based on complex physical principles, the statistical models use a function of instantaneous vehicle states. For example, the VT-micro model, developed first by Virginia Tech using the Portable Emission Measurement System (PEMS) data, estimates emission by following equations:

$$MOE_e = \begin{cases} exp(\sum_{i=0}^{3} \sum_{j=0}^{3} (L_{i,j}^e \cdot v^i \cdot a^j)) & a \geq 0 \\ exp(\sum_{i=0}^{3} \sum_{j=0}^{3} (M_{i,j}^e \cdot v^i \cdot a^j)) & a < 0 \end{cases} \tag{28}$$

where MOE_e is the instantaneous fuel consumption or emission rate of a pollutant specie e; e can be CO, HC and NOx; v and a represent the instantaneous vehicle speed(km/h) and acceleration(km/h/s) separately; $L_{i,j}^e$ and $M_{i,j}^e$ are the regression coefficients. A logarithm transform is adopted to avoid negative output and to enhance the model representativeness in low-speed and/or low-acceleration regimes. The model was created for positive and negative accelerations separately to ensure a better compliance with the measurement data over the full range of the vehicle-operation envelope. Besides using PEMS data, a method was developed to calibrate the model using aggregated emission information before being applied in the context of European roads [16].

III. NUMERICAL EXAMPLE

In this section, we illustrate practical examples that the method presented above is applied. Let's consider the basic problem for local fleet management application, where control of vehicle trajectory is necessary for the purpose of managing local emissions, such as carbon footprint (CO_2) and HC.

In the former study, four main scenarios were proposed when deriving optimal driving strategy for single vehicle fuel

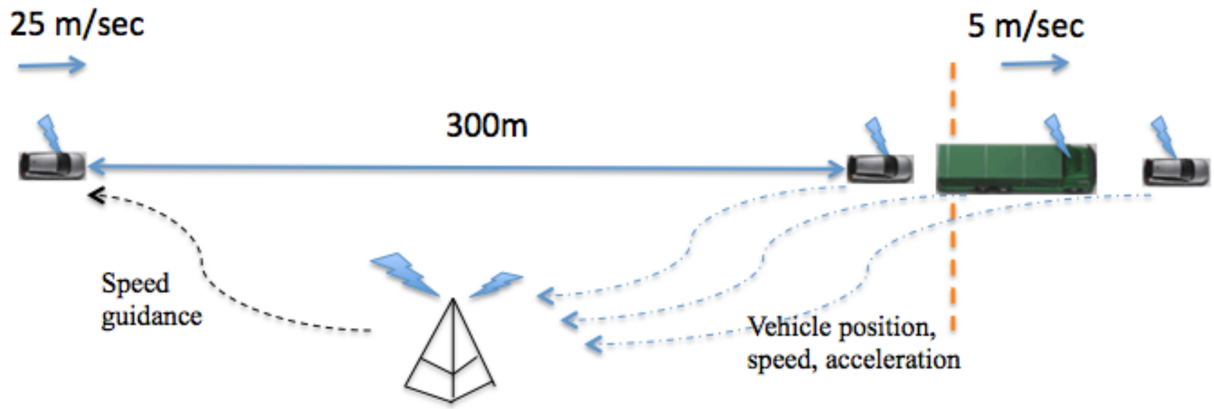

Figure 2. An example of the fixed boundary optimal control problem.

economy [9]. Here we simply consider a most common scenario when vehicles in a fleet meets congestion downstream on highway. A central node of road-side unit can detect the congestion formulation beforehand via communication and other sensors, and then estimate current traffic state downstream, traffic flow speed, based on traffic models. Thus, the fleet management system needs to derive the optimal speed trajectory for vehicle or vehicle fleet, which can be disseminated to drivers. When the congestion is disseminated, vehicle may accelerate to a desired speed. This involves an acceleration process during which some harmful emissions produces (e.g. HC and CO). Figure 2 shows a detailed case of the scenario with parameters. Assume the downstream traffic flow speed is in a heavy congested mode with a speed level of 5 m/sec while a vehicle travels at a speed of 25 m/sec upstream. There is a distance of 300m from the current position of the vehicle to the downstream traffic flow. The goal is to find a speed profile and control inputs that minimize the total carbon produced in the procedure. For the acceleration process after congestion dissemination, the vehicle needs to recover to a speed of 20 m/sec from current low speed.

Applying the dynamic programming approach presented by the equation (27) and the instantaneous emission model of the equation (28), we can estimate a trajectory that fulfill the minimal CO_2 emission for the vehicle using backward recursion and forward recovery technique [12]. The same is conducted for acceleration process whereas the HC emission is concerned instead. A space discretization of 5 m is applied in the solver. Figure 3 shows the estimated speed state profiles against space and time respectively. The whole process takes about 15 seconds (45 seconds for acceleration). Emission models for a light duty truck is applied. The picture shows that the optimal strategy in this example is to keep the speed for a certain time interval (close to 6 seconds) and then conducting a smooth brake, which represents almost a constant deceleration about $-2m/sec^2$. This may indicate that the objective function on CO_2 is locally close to a convex function during braking. Given the speed range and distance, a constant speed combing with a smooth deceleration leads to minimal carbon footprint. On the other hand, the acceleration process that produces minimal HC emission seems quite nonlinear.

IV. Summary and further perspectives

In this paper we first illustrate the opportunities that cooperative systems based on V2I communication have brought to dynamic traffic management applications. The focus is mainly on the transport energy and induced negative emission impact problems. The study shows an important step in developing an intelligent fleet management system using V2I infrastructure able of minimizing local fuel consumption or other emissions produced by fleet. Inspired by the early work in optimal fuel economy driving, we propose to solve the optimal trajectory problem by applying the optimal control theory. In particular, a discrete dynamic programming formulation is used while the formulation is based on a simpler space discretization. Instantaneous emission model is applied in the calculation of the objective function using vehicle states. In the case study, a most common traffic scenario on highway, downstream congestion formulation (backward shockwave), is considered. By applying standard technique of backward recursion and forward recovery, ptimal speed profiles and corresponding control inputs are derived for minimal carbon footprint in braking and minimal HC for acceleration process.

Obviously, there is still a large space to extend the study for intelligent fleet management application. At the moment, only a single vehicle is treated when deriving optimal trajectory. When a fleet of heavy vehicles is considered, it is necessary to treat the fleet as an entity that can interact with other vehicle and fleets. Since the ACC control within vehicle fleet has been already developed in several studies such as [6], the current work is to derive the optimal trajectory for a fleet where the vehicles are modeled as a whole.

Besides the derivation of the fleet control strategies in different traffic scenarios, it is also important to test the performance of the derived trajectory control approach. Such

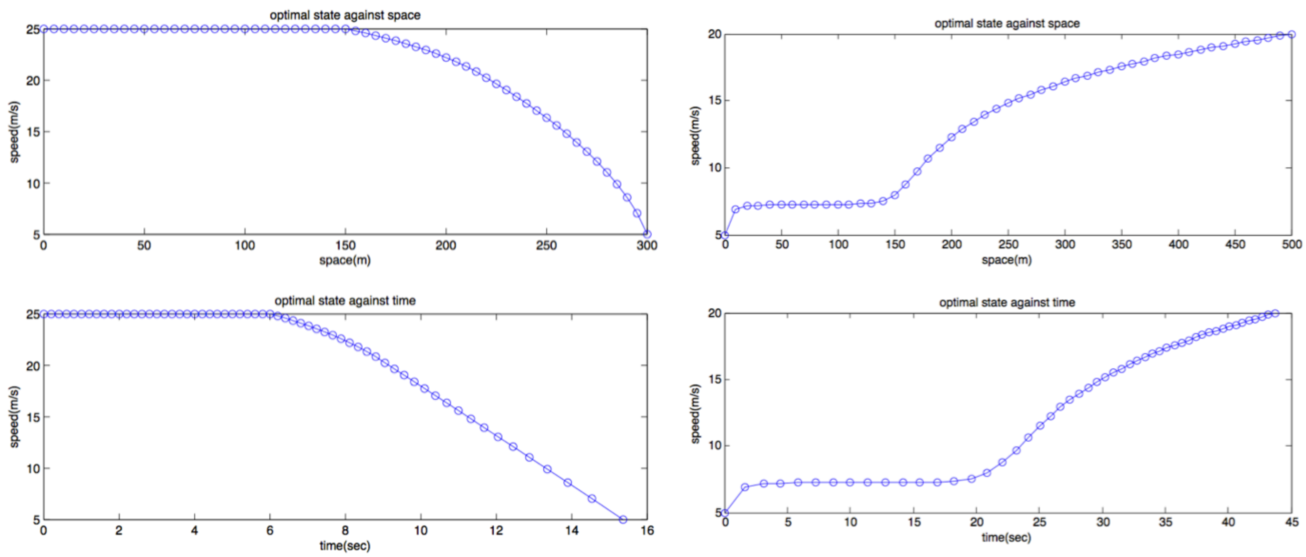

Figure 3. A speed profile optimal for CO2 in braking (left) and a speed profile optimal for HC in acceleration.

evaluation requires real traffic situations and interaction with other traffic objects such as traffic light, traffic sign etc. Therefore, evaluation of the control approach using traffic simulation is indispensable during the development of the dynamic traffic management system.

ACKNOWLEDGMENTS

The author would like to thank the financial support of the iQFleet project by the Swedish Innovation Agency (VINNOVA).

REFERENCES

[1] European Commission, "Roadmap to a single european transport area - towards a competitive and resource efficient transport system," White Paper COM(2011) 411 final, 2011.

[2] World Business Council for Sustainable Development, "Mobility 2030: Meeting the challenges to sustainability," 2004.

[3] F. Browand, J. McArthur, and C. Radovich, "Fuel saving achieved in the field test of two tandem trucks," California PATH research report, Tech. Rep. UCB-ITS-PRR-2004-20, 2004.

[4] A. Alam, *Fuel-Efficient Distributed Control for Heavy Duty Vehicle Platooning*. Stockholm, Sweden: Licentiate thesis TRITA-EE 2011:059, Royal Institute of Technology, 2011.

[5] P. Sahlholm, "Distributed road grade estimation for heavy duty vehicles," Ph.D. dissertation, KTH, Automatic Control, 2011.

[6] K. Liang, "Linear quadratic control for heavy duty vehicle platooning," Master's thesis, Royal Institute of Technology (KTH), Sweden, 2011.

[7] H. Farah, H. Koutsopoulos, M. Saifuzzaman, R. Kölbl, S. Fuchs, and D. Bankosegger, "Evaluation of the effect of cooperative infrastructure-to-vehicle systems on driver behavior," *Transportation Research C*, vol. 21, pp. 42–56, 2012.

[8] J. Hooker, A. Rose, and G. Roberts, "Optimal control of automobiles for fuel economy," *Transportation Science*, vol. 17, no. 2, pp. 146–167, May 1983.

[9] J. Hooker, "Optimal driving for single-vehicle fuel economy," *Transportation Research A*, vol. 22A, no. 3, pp. 183–201, 1988.

[10] A. Bryson and Y. Ho, *Applied Optimal Control: Optimization, Estimation and Control*. Taylor & Francis, 1975.

[11] L. C. Evans, "An introduction to mathematical optimal control theory," Lecture notes, Dept. of Mathematics, University of California, Berkeley.

[12] U. Jönsson, C. Trygger, and P. Ögren, "Optimal control," Lecture notes, Optimization and system theory, Dept. of Mathematics, Royal Institute of Technology (KTH), Sweden.

[13] X. Ma, W. Lei, I. Andréasson, and H. Chen, "An evaluation of microscopic emission models for traffic pollution simultion using on-board measurement," *Environmental Modeling & Assessment*, vol. 17, no. 4, pp. 375–387, August 2012.

[14] M. Barth, F. An, T. Younglove, G. Scora, C. Levine, M. Ross, and T. Wenzel, "Development of a Comprehensive Modal Emissions Model," National Cooperative Highway Research Program, Final Report Project 25-11, 2000.

[15] H. Rakha, K. Ahn, and A. Trani, "Development of VT-Micro model for estimating hot-stabilized light duty vehicle and truck emissions," *Transportation Research Part D*, vol. 9, pp. 49–74, 2004.

[16] X. Ma, Z. Huang, and H. Koutsopoulos, "A numerical approach on model calibration for an emission simulation platform," in *proceedings of the 90th Transportation Research Board Annual Meeting,*, 2011.